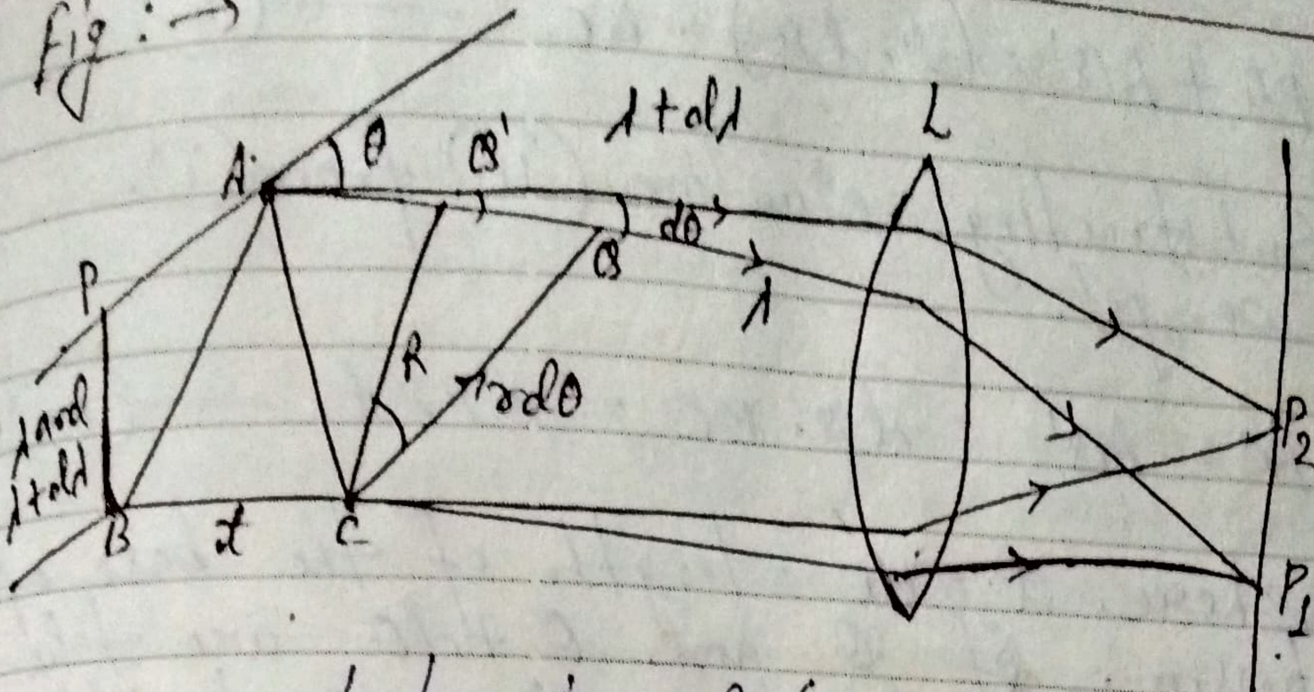
9 · Expression for resolving power of
10 a prism :→
· Let a parallel beam of
11 light consisting of wave lengths
· λ and $\lambda + d\lambda$ be refracted through
12 a prism which is placed in the
· position of minimum deviation. BP is
13 the incident plane wave front and
· CQ and CQ' are the emergent wave
fronts corresponding to wave length

λ and $\lambda + d\lambda$ respectively. L is the objective of the telescope of a spectrometer. P_1 and P_2 are the positions of the central maxima of diffraction pattern's of two spectral lines of wave lengths λ and $\lambda + d\lambda$ respectively.

fig :→



Let μ and $(\mu - d\mu)$ be the refraction indexes corresponding to wavelength λ and $\lambda + d\lambda$ respectively.

According to Fermat's principle the actual optical path between the incident and emergent wave fronts for any wave length must be the same.

Thus, for wave length λ, we have,

$$PA + AQ = \mu \cdot BC \quad \text{—— (1)}$$

Since the path BC in the prism is equivalent to $\mu \cdot BC$ in air, and for wave length $\lambda + d\lambda$, we have.

$PA + AQ' = (r \cdot dr) \cdot BC$ —— (ii)

Substracting equation (ii) from (i).
we get,

$AQ - AQ' = dr \cdot BC = dr \cdot t$ —— (iii)

where $t = BC$ = length of the base of prism. If θ and $\theta + d\theta$ are the angles of deviation for wavelength λ and $\lambda + d\lambda$ respectively.
we have,

$\angle Q'AQ = \angle QCR = d\theta$

Therefore from fig, we have $AQ' = AR$
so that,

$AQ - AQ' = AQ - AR = RQ$ —— (iv)

Comparing (iii) and (iv), we get,

$RQ = dr \cdot t$

But, $RQ = CQ \cdot d\theta = a \cdot d\theta$.

(where $CQ = a$)

$\therefore a\,d\theta = d\lambda \cdot t$

or, $\theta = \frac{t}{a} \cdot d\lambda$ —— (V)

This equation gives the angular separation of the two wavelengths. As the emergent beam has a rectangular cross section, the prism may be considered as a rectangular aperature of width 'a' which is equal to the width of the emergent beam. The angular half width, i.e, the angle $d\theta_1$ between central maximum and first minimum of λ due to a rectangular apereture of width 'a' is given by,

$$a \sin d\theta_1 = \lambda$$

or, $d\theta_1 = \lambda/a$ —— (VI)

(As $d\theta_1$ is small, $\therefore \sin d\theta_1 = d\theta_1$)

According to Rayleighs criterion for just resolution, the two spectral lines are

Just reached when the central maximum (R_2) of second spectral line of wave - length $\lambda + d\lambda$ will lie to the first secondary minimum of first spectral line of wave length λ, this condition implies that for just resolution the angle between minimum of either diffraction pattern must be equal to the angular separation of two wave - lengths.

Thus, for just resolution,

$$d\theta = d\theta_1$$

or, $\frac{t}{a} \cdot d\mu = \frac{\lambda}{a}$

or, $\lambda = t \cdot d\mu$.

$\therefore$ The (spectral) resolving power of the prism,

$$\frac{\lambda}{d\lambda} = t \cdot \frac{d\mu}{d\lambda} \text{ ——— (VII)}$$

Thus, the spectral resolving power of a prism (i) is directly proportional to the width of the base of the prism

(11) is directly proportional to the rate of change of refractive index of the material of the prism with the wave-length.

The — End